**AY C10/L&S C70U** **Name:** ____________________

**Fall 2018**

**Worksheet 5**

**09/18/2018**

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This worksheet covers the physical intuition behind the greenhouse effect, tides, and extrasolar planet detection. There are 4 pages (including this cover page) and 3 questions. Accuracy is definitely desired, but effort and clear physical reasoning are far more important than the final answer, *especially* for the challenge problems.

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1. Consider a simplified model for the atmosphere of Venus, a flat plate that is completely transparent to light with a wavelength shorter than 1 micrometer, but which reflects *half* of all incident light with longer wavelengths.

   (a) **Would the incident sunlight** be allowed to pass through the atmosphere, or would it be reflected? How about **Venus's thermal emission**? Answer this question with minimal calculation. The incident sunlight is emitted primarily in the visible, so it would be allowed through the atmosphere—however, since Venus has a lower equilibrium temperature than the Sun, the resulting thermal emission would peak in the infrared and so would be blocked by the atmosphere.

   (b) Assume that the Sun is very far away and its rays come in parallel. **Draw a figure** illustrating the path of the light. **What happens to it** when it hits Venus's surface initially and how does that affect its subsequent trajectory? The path of the light was drawn in section.

   When the Sun's rays hit venus, they are absorbed and reradiated as infrared light. This causes the light to be trapped by Venus's atmosphere.

   (c) In order to remain in thermodynamic equilibrium, the total radiative flux ($\varepsilon$ in the lecture notation) absorbed by each square meter of the planet must equal the flux it emits. With our greenhouse atmosphere, **how does $\varepsilon$ change** with respect to the no-atmosphere case? **Temperature**? Answer qualitatively.
   The radiative flux $\varepsilon$ is greater with the greenhouse atmosphere than without; since in thermal equilibrium $\varepsilon = \sigma_{\mathrm{SB}} T_{\mathrm{eff}}^4$, the temperature also increases due to the greenhouse atmosphere.

   (d) If instead of reflecting 50% of infrared light, the atmosphere blocked 10%, **what would happen** *(qualitatively)* to the change in temperature? **5%**? Use this result and whatever facts you know from lecture and reading to order the four inner planets (Mercury, Venus, Earth, Mars) by the strength of their greenhouse effect. Intuitively, the more visible-transparent but infrared-blocking substance (greenhouse gas) is in the atmosphere, the greater the equilibrium temperature would be. Thus, the greenhouse effect would be weakest on Mercury (with almost no atmosphere), increasing in strength on Mars (with a thin $CO_2$ atmosphere), Earth (with $CO_2$ and vater vapor), and finally Venus (with a $CO_2$-driven runaway greenhouse effect).

(e) **Explain in a sentence or two** how adding carbon dioxide to the atmosphere may contribute to global warming on Earth. Carbon dioxide is a greenhouse gas because it is opaque in regions of the infrared while being transparent to visible light. Pumping more carbon dioxide into the atmosphere will not block any incident sunlight, but will trap more of the infrared radiation that the Earth re-radiates and so raise its temperature.

2. For this question, we investigate tidal forces using the example of a small moon in the orbit of Saturn. The satellite has a mass $m$ and a radius $r$, and orbits around Saturn (a planet of mass $M$) at radius $R$.

(a) **Draw a diagram** of the system, including accelerations from Saturn on both sides of the moon and the acceleration at the moon's center-of-mass.

(b) **Compute the accelerations** at each point you indicated in the previous diagram. Now **subtract the center-of-mass acceleration** off of all accelerations in your picture. **Qualitatively compare** the *magnitudes* and *directions* of the accelerations on either side of the moon. **Describe** what effect you would expect this to have on the moon's shape. The total acceleration at all points is in the direction of Saturn, stronger than the center-of-mass acceleration on the side closer to Saturn and weaker on the side further away. Relative to the center of mass, this means that the closer-in side is pulled inward while the farther-away side is pushed outward, resulting in the body being stretched out.

(c) Besides being affected by tidal acceleration, the shape of the moon is also influenced by its own self-gravity. **Equate the acceleration due to self-gravity** with the tidal acceleration. **What happens** when tidal acceleration equals or exceeds self-gravitational acceleration, and what course concept does this connect to? When the tidal acceleration at the surface of the moon exceeds the acceleration from self-gravity that holds the moon together, the moon must necessarily be torn apart. The greatest radius around a planet (in this case, Saturn) at which this occurs is the *Roche lobe*, a concept mentioned in lecture.

(d) Very qualitatively, **how might one adjust** $m$, $M$, $r$, or $R$ to cause tidal disruption? **How does tidal disruption** explain why Saturn has rings? Increasing the mass of the moon $m$ raises its self-gravity, while increasing $R$ decreases the magnitude of tidal forces—increasing both would help stabilize the moon against tidal disruption. By contrast, raising $r$ alone (without changing $m$) would decrease the strength of self-gravity while raising $M$ would increase the strength of the tidal force, making an object more liable to disruption.

Saturn's moons can be explained by an object moving inward slowly, past the distance $R$ at which an object of its mass $m$ and radius $R$ would be tidally disrupted. Upon disruption this object broke into much smaller objects, which still orbited Saturn—which we observe now as rings.

(e) The *Cassini* space probe passed very close to Saturn and has a much smaller mass than any of Saturn's moons, and yet survived without being tidally disrupted. **Why**? Because of its small mass, one would expect *Cassini* to be tidally disrupted even at great distances from Saturn. However, the Roche limit analysis for tidal disruption applies only to objects held together by self-gravity—*Cassini* is held together by intermolecular interactions. This is the same reason why you and I don't break apart despite being well within the Earth's Roche lobe for objects of typical human size and mass!

3. In this question, we investigate the two main methods of exoplanetary detection—the Doppler-wobble ("radial velocity") and transit techniques.

(a) Suppose that a planet is orbiting a star of mass $M$ at distance $R$, with peak velocity $v_0$. Assuming we are observing the star edge-on, **plot the Doppler wobble** in light of wavelength $\lambda$ that we would measure as a function of time. Assuming a circular orbit, the observed Doppler wobble would follow the functional form

$$\Delta\lambda = \lambda v_0/c \cos(2\pi t/P) \tag{1}$$

where $P = 2\pi\sqrt{R^3/GM}$ is the measured orbital period of the planet. This is because at some times the motion is toward/away from us along our line of sight (radial), whereas at other times it is parallel to the celestial sphere and not along our line of sight (tangential). Throughout its orbit the motion of the planet has both radial and tangential components.

The only observable quantities here are $\Delta\lambda$ and $P$; we can estimate that $v_0 = (\Delta\lambda/\lambda)c$. With these quantities it is possible to infer the masses of the planet and star.

(b) Now let the the plane of the planet's orbit be actually inclined by a (not necessarily small) angle $\alpha$ with respect to our line of sight. **Modify your expression from the previous part** to incorporate the inclination and **qualitatively describes** what happens to the Doppler wobble as we see a planet's orbit more and more face-on. The Doppler wobble picks up the additional factor of $\sin\alpha$ (also denoted as $\sin i$ in lecture) and so becomes

$$\Delta\lambda = \lambda v_0/c \cos(2\pi t/P) \sin\alpha \tag{2}$$

Qualitatively, this means that the magnitude of the Doppler wobble peaks when the planet's orbit is seen edge-on (when $\alpha = 90°$), and falls to zero when the planet's orbit is seen face-on. This is because when the planet's orbit is face-on, there is no radial component to the velocity that would create a Doppler wobble. However, this is a gentle roll-off that still allows a planet to be detected across a wide range of inclinations.

(c) Now assume the star has a radius of $r_*$ and the planet has a radius $r_p$. If the planet is orbiting edge-on, **sketch out the shape** of a transit. At transit peak, **what fraction of starlight** would be blocked? At the peak of a transit, the observed flux would reduce by a factor $A_{\mathrm{cross-section,planet}}/A_{\mathrm{cross-section,star}} = (\pi r_p^2)/(\pi r_*^2) = (r_p/r_*)^2$ would be blocked by the planet, since the star emits at roughly constant intensity across its surface.

(d) Transits can only be detected when the plane of the planet's orbit is edge-on or nearly so. **Explain** how this makes transiting planets comparatively rare. The inclination of planetary orbits is essentially randomly distributed, but we can only see those whose orbits are edge-on—specifically, those for which the planet appears to occult some part of the star. For a planet orbiting at radius $R$, this occultation occurs when the inclination$\alpha \approx \pi/2 \pm (r_* + r_p)/R$. Because $r_*, r_p \ll R$, the range of inclinations over which a planet crosses its star (from Earth's point of view) is small and so planetary transits are in one sense less detectable than their radial velocities.