AY C10/L&S C70U **Name:** ______________________

Fall 2018

Worksheet 6

10/14/2018

This worksheet covers the Sun and the stars. There are 5 pages (including this cover page) and 2 questions. Accuracy is definitely desired, but effort and clear physical reasoning are far more important than the final answer.

---

1. In this problem, you will answer a variety of short questions about the Sun, stars, and star formation.

   (a) The luminosity of the Sun $L_\odot = 3.828 \times 10^{26}\,\mathrm{W}$, while the human eye's pupil has a radius of $0.2\,\mathrm{cm}$. If the weakest signal the human eye can discern is 100 photons/s, **compute the greatest distance from Earth** at which an exact copy of the Sun would be visible with the naked eye. As a simplifying assumption, assume that all of the star's light is emitted as $500\,\mathrm{nm}$ photons. The power that we would receive from this Sun-like star at a distance D would be distributed over a sphere of that radius, so to compute the power per unit area, we would just use

$$F = \frac{P}{A} = \frac{L_\odot}{4\pi D^2} \tag{1}$$

Recall that the power P is just the energy released per unit time, $P = \Delta E/\Delta T$. In this case, the energy is emitted (and received) as photons of energy $E_{\mathrm{phot}} = hf$—consequently, the total $\Delta E = \Delta N E_{\mathrm{phot}} = \Delta N(hf)$.

In the context of this problem we care about the total rate of photons emitted, that is r_{phot}:

$$r_{\mathrm{phot}} = \frac{\Delta N}{\Delta T} = \frac{\Delta E}{\Delta T}(hf)^{-1} = \frac{P}{hf} = \frac{L_\odot}{hf} \tag{2}$$

These photons are spread over a sphere of radius D, so the number of photons per unit area at that distance is just:

$$F_{\mathrm{phot}} = \frac{L_\odot}{4\pi D^2 hf} \tag{3}$$

But how many photons does the human eye collect from the star? In this case, that would be the number of photons per unit area, times the area of the pupil (a circle of radius r_{pupil}. So we find that the rate of photon collection by the human eye c_{phot} is:

$$c_{\mathrm{phot}} = F_{\mathrm{phot}} A_{\mathrm{pupil}} = \frac{L_\odot \pi r_{\mathrm{pupil}}^2}{4\pi D^2 hf} = \frac{L_\odot r_{\mathrm{pupil}}^2}{4D^2 hf} \tag{4}$$

The end goal of this problem is to find the greatest distance from the Earth at which a star would be detectable, given some threshold $c_{\mathrm{phot,thresh}} = 100\mathrm{photons/s}$—that is, we need to isolate D on one side. This yields that

$$D_{\text{thresh}}^2 = \frac{L_\odot r_{\text{pupil}}^2}{4hfc_{\text{phot,thresh}}} \to D_{\text{thresh}} = \sqrt{\frac{L_\odot r_{\text{pupil}}^2}{4hfc_{\text{phot,thresh}}}} \tag{5}$$

Substituting in the values given at the start of the problem, and the value of Planck's constant 6.67×10^{-34} J s, we find that $D_{\text{thresh}} \approx 100$ parsec. At distances greater than D_{thresh}, the c_{phot} that we would receive would be lower than the threshold $c_{\text{phot,thresh}}$ quoted in the problem, so the star would be undetectable; at or closer to that distance, however, the photon rate is equal to or greater than the threshold for detector and so we can see it.

(b) The luminosity of a star typically scales with its mass as $L \propto M^4$. Use this scaling relation to answer the following subparts:

i. **Estimate the luminosity** of a star of $3M_\odot$. For all scaling problems, there has to be a scaling point—in this case, we use the Sun's mass and luminosity as a scaling point, since we know that a star of $1M_\odot$ should have a luminosity of $1L_\odot$. This yields the equation

$$\left(\frac{L_*}{L_\odot}\right) = \left(\frac{M_*}{M_\odot}\right)^4 = \left(\frac{3M_\odot}{M_\odot}\right)^4 = 81$$
$$L_* = 81L_\odot \tag{6}$$

ii. Throughout its main-sequence lifetime, a star will burn approximately 10% of its initial mass. This process will give the Sun a main-sequence lifetime of approximately 11 billion years. **Derive a scaling relation** for stellar lifetime, and use it to **estimate** the lifetime of the $3M_\odot$ star. The lifetime that a star lives increases in proportion to the material that a star has to burn over its lifetime, but decreaes with increasing luminosity. This allows us to construct a scaling relation for the star's lifetime τ_*:

$$\tau_* \propto M_{\text{burn},*} L^{-1} \tag{7}$$

We know that $M_{\text{burn},*} = 0.1M$, and $L_* \propto M_*^4$, so substituting these into the scaling relation, we get that

$$\tau_* \propto (0.1M_*)(M_*)^{-4} \propto M_*^{-3} \tag{8}$$

Where the 0.1 is dropped because it is just a constant.

Using the Sun's lifetime as a scaling point, we can convert this proportionality into an equation:

$$\left(\frac{\tau_*}{\tau_\odot}\right) = \left(\frac{M_*}{M_\odot}\right)^{-3} = (3)^{-3} \to \tau_* = \tau_\odot/27 = 4.1 \times 10^8 \text{ y} \tag{9}$$

2. A *Hertzsprung-Russell diagram* (HR diagram) plots the temperature and luminosity of a list of stars. Use the HR diagrams below to answer the following questions:

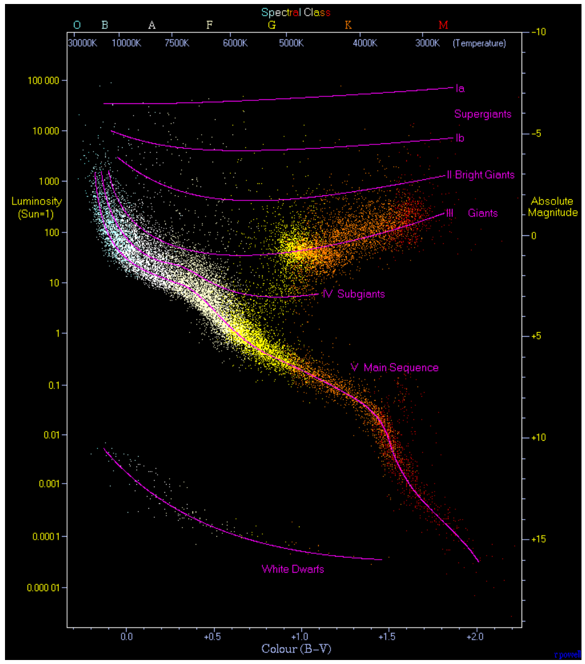

Figure 1: The HR diagram for the all-sky *Hipparcos* survey.

(a) **Briefly explain** the significance and characteristics of O, B, A, F, G, K, and M spectral types. There is no need to go into detail on each one; just discuss general trends. Broadly speaking, O stars have the hottest and bluest surfaces—the surface temperature then falls as one passes through B, A, F, G, and K stars, eventually reaching the relatively cool M stars. On the main sequence, O-type stars tend to be the most massive and luminous while M-type stars are the least—however, red giant stars break this trend, because although their surfaces are relatively cool, they are very large and so have a much larger surface area over which to radiate.

(b) Stars of more than 2 $M_\odot$ have short enough lifetimes that within the age of the Universe, there have been many stages of star formation and death. Approximately 15% of such are observed to be giants. **Explain briefly** why this is the case. Because there have been many cycles of star formation and death, we essentially see a representative sample of all life stages of these stars. That 15% of stars in this mass range are giants tells us that they spend approximately that fraction of their lifetime on the giant branch.

(c) As the Universe grows older, **what would you expect** to happen to the number of stars on the white-dwarf branch?
As the Universe grows older, low-to-intermediate-mass stars currently on the main sequence will pass through the stages of red giant, planetary nebula, and white dwarf. Moreover, new stars will form and eventually undergo the same progression. This will increase the number of stars on the white-dwarf branch.

(d) The probability of a star forming on the main sequence at a particular mass is given by $\eta(m) = Am^{-2.35}$, so in principle we would expect most of the stars we observed to be on the lower-mass, lower-luminosity end of the spectrum. **Why, then,** does the *Hipparcos* HR diagram have more stars at high mass and high luminosity? **Justify** using a scaling argument. *(Hint: generalize the result about farthest detection distance of Sunlike stars.)* As demonstrated with the toy example in part 1(a), the inverse-square law (combined with a hard cutoff for detectability) means that the distance $D_{\text{thresh}}(L)$ at which we can detect stars decreases with decreasing luminosity. The volume in which we can see stars of that luminosity is thus $V_{\text{thresh}} = (4\pi/3)D_{\text{thresh}}(L) \propto D_{\text{thresh}}(L)^3$.

This means there's a *far* smaller volume of space in which we can see lower-luminosity stars than higher-luminosity stars—an effect that in the case of *Hipparcos*, overcomes the fact that giant stars are intrinsically rarer to result in an HR diagram with a disproportionate number of O/B/A-type and giant-branch stars.

(e) We can construct HR diagrams not only for the whole sky, but for individual star clusters as well. In the following subparts, we qualitatively analyze the following HR diagram for a single star cluster. *(Note: The x-axis represents a color index [an observational proxy for temperature] for temperature whereas the y-axis represents the luminosity in absolute magnitude. How they translate to one another is beyond the scope of this course.)*

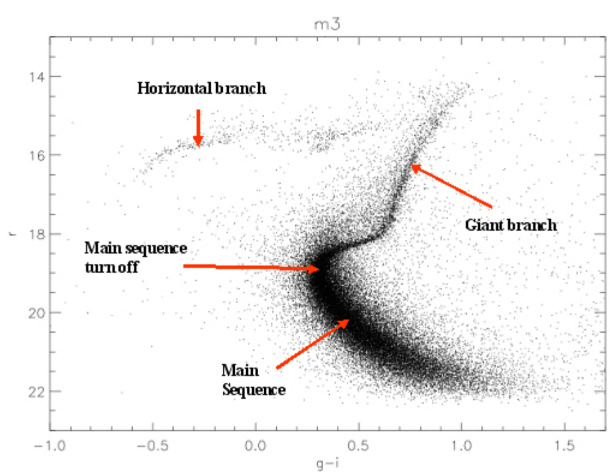

Figure 2: An HR diagram for a single star cluster. The x and y scales are proxies for temperature and luminosity.

i. Unlike the whole-sky HR diagram from Hipparcos, the cluster HR diagram has a clear main-sequence turn-off. Using what you know about star clusters, **explain the origin** of this feature. **How could we use it** to determine the age of a cluster?
The main-sequence turn-off is a consequence of the fact that all stars in a clus-

ter formed at the same time (although occupying a wide range of mass). As a consequence, all stars of a particular mass (and main-sequence luminosity) will be peeling off of the main sequence onto the giant branch at the same time.

Knowing the luminosity of these stars allows us to know their mass and main-sequence lifetime—because these stars are at the end of their main-sequence lifetime and they must have (by definition) formed with the whole cluster at the beginning, we can infer that the main-sequence lifetime of the stars that are cutting off the main sequence is the age of the cluster.

ii. In the long run, **what would you expect to happen** to the main-sequence branch of this cluster?

In the long run, longer-lived (and thus, less massive, less luminous, and redder) stars will turn off the main sequence, causing the main-sequence turnoff to shift downward and to the right on an HR diagram. At very, very long times, all stars in that cluster will have fused all their core hydrogen and evolved off of the main sequence and so the main-sequence branch will be empty.