AY C10/L&S C70U **Name:** ____________________

Fall 2018

Worksheet 7

10/13/2018

This worksheet focuses on the Sun and the stars. There are 5 pages (including this cover page) and 4 questions. Accuracy is definitely desired, but effort and clear physical reasoning are far more important than the final answer, *especially* for the challenge problems.

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1. In this problem, you will answer a few short questions about the Sun.

   (a) When stars form out of molecular clouds, their masses follow a distribution similar to that shown below (for the nearby Pleiades cluster). **Indicate the location of the Sun** on the plot—based on your result, do you think that the "typical" star has a mass **larger, lower, or equal** to that of the Sun?

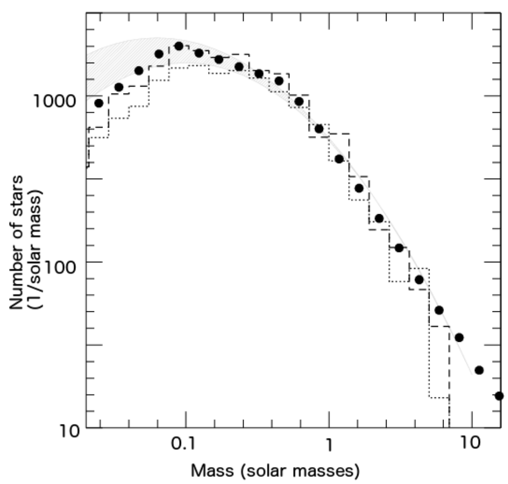

Figure 1: Number of stars at various masses in the Pleiades cluster.

The "typical" star has a **lower** mass than the Sun because most of the stars on the plot appear to be below 1 $M_\odot$.

   (b) The Sun has the following layers, in no particular order: photosphere, corona, chromosphere, radiative zone, convective zone, core. **Sort the layers** in order of increasing distance from the Sun's center. If you were to look straight at the Sun on a typical day, **which of these layers** would you see? core, radiative zone, convective zone, photosphere, chromosphere, corona. We see the **photosphere** on a typical day (but the corona when the photosphere is blocked by a total eclipse.)

   (c) In one or two sentences, **explain the process** of convection. In convection, hot gas rises and cool gas falls, resulting in the transport of energy upward. In the Sun, this process causes granules to form on the surface.

2. Stars are powered by *nuclear fusion* of various elements in their cores; main-sequence stars in particular are supported by fusion of hydrogen into helium.

   (a) **Explain**, in one or two sentences, the significance of Einstein's formula $E = mc^2$ in the context of nuclear fusion.Einstein's formula demonstrates that mass and energy can be converted into one another. In any given reaction (including fusion reactions), the energy released $\Delta E = (m_{\text{reac}} - m_{\text{prod}})c^2 = (\Delta m)c^2$. If the reactants are more massive than the products then the reaction releases energy; if less massive, then the reaction absorbs energy from the surroundings.

   (b) The mass of the product in each hydrogen-to-helium fusion reaction is $6.644\,24 \times 10^{-27}$ kg while that of the reactants is 6.7616×10^{-27} kg. **Compute the energy released** by each reaction.
   In this case, we have that

$$\begin{aligned} \Delta E &= (m_{\text{reac}} - m_{\text{prod}})c^2 \\ &= (6.7616 \times 10^{-27}\,\text{kg} - 6.644\,24 \times 10^{-27}\,\text{kg})(3 \times 10^8\,\text{m}\,\text{s}^{-1})^2 \\ &= 1.05 \times 10^{-11}\,\text{J} = 6.58 \times 10^7\,\text{eV} \end{aligned} \tag{1}$$

   (c) Suppose this reaction releases all of its energy as a single photon. **Compute its frequency**. **Is this frequency comparable** to what we observe at the Sun's surface, and why or why not? The frequency in this case can be found from the formula $E_{\text{phot}} = hf \to f = E_{\text{phot}}/h$. Because all the energy of fusion is released as a single photon, we know that $E_{\text{phot}} = \Delta E$ from the previous problem; substituting in that Planck's constant $h = 6.67 \times 10^{-34}\,\text{J}\,\text{s}$, we can find that the frequency $f = 1.59 \times 10^{22}\,\text{Hz}$.

   This is *much higher in frequency* than the (typically) visible light from the Sun's surface—because on its way to the surface, this photon is converted into random thermal motion before finally being released as Planck radiation at the photosphere.

   (d) Assume the Sun was born with a mass of 1.981×10^{30} kg, all pure hydrogen, and fuses 10% of its mass into helium on the main sequence.

      i. Using your result from the previous part, **how much total energy** does the Sun emit over its lifetime? How many fusion reactions will take place over the lifetime of the Sun, and how much energy is released per reaction? To compute this we divide the total mass fused over the Sun's lifetime M_{fused}, by the mass of the reactants in each reaction, and multiply by the energy per reaction:

$$E_{\text{lifetime}} = \frac{M_{\text{fused}}}{m_{\text{reac}}}\Delta E_{\text{fusion}} = \frac{0.1 M_\odot}{m_{\text{reac}}}\Delta E_{\text{fusion}} = 3.15 \times 10^{44}\,\text{J} \tag{2}$$

      Note especially that this is *not* 10% of the Sun's mass being converted into energy via $E = mc^2$—rather, 10% of the Sun's mass undergoes the hydrogen-to-helium fusion reaction (which releases energy equivalent to about 0.7% of the reactant mass). Thus, the Sun will only lose about $(0.1)(0.007) \to 0.07\%$ of its total mass as energy while on the main sequence.

ii. With a luminosity of $L_\odot = 3.84 \times 10^{26}$ W, **estimate the Sun's main-sequence lifetime**. Recall that luminosity is a power (energy per unit time).Over its lifetime, the Sun radiates an energy E_{lifetime} with a rate of $L_\odot = \Delta E/\Delta T$. By dimensional analysis, we can convert this into a lifetime of the Sun using

$$\tau_\odot = \frac{E_{\text{lifetime}}}{L_\odot} = 2.60 \times 10^{10}\,\text{y} \tag{3}$$

This figure is 2-3 times as high as the actual predicted main-sequence lifetime of the Sun, but then again this was a back-of-the-envelope calculation using round numbers like 10% of mass burned, and ignoring the fact that the Sun's luminosity will increase as it reaches the end of its main-sequence lifetime.

These and other factors affecting the Sun's main-sequence lifetime are all beyond the scope of the class—but even ignoring those, we got a decent order-of-magnitude estimate (and scaling other stars off of the real solar lifetime of roughly 10 billion years should give results accurate to within 10-20%).

(e) Once a very massive star starts fusing iron into heavier elements, it rapidly experiences core collapse and explodes as a supernova. **Explain physically** why this is the case.
A plot of masses per nucleon (or "binding energy") shows that the mass of each nucleon in an element declines with increasing atomic number, until reaching iron—after that, the mass increases with increasing atomic number.

Iron has the lowest mass per nucleon, so fusing all the elements up to iron means that the products have *lower* mass than the reactants and the reaction releases energy; by contrast, fusing elements more massive than iron means that the products have a *greater* mass than the reactants and thus the reaction absorbs energy. In a star that is fusing iron into heavier elements, this energy is the internal thermal energy of the star—the loss of this energy causes the star's core to shrink and heat up even further, eventually culminating in core collapse and a supernova.

Note: *Ultimately, the star's core becomes supported primarily by electron degeneracy pressure rather than thermal (gas) pressure, and when the core grows so massive that even that can no longer support it, it undergoes a rapid* core collapse *(into a neutron star or black hole) which causes the star to explode as a supernova.*

3. In this question, we review various physical concepts with the example of a 1 $M_\odot$ and 10 $M_\odot$ star in a binary system. The stars are separated by a distance of 1000 AU.

(a) **Draw a diagram** of the system, indicating the position and distance between the stars. **Which star** would you expect to be closer to the center of mass of the system, and why?
The 10 $M_\odot$ star is closer to the center of mass of the system, because by the definition of a center of mass, $m_1(x_{1,\text{cm}}) = m_2(x_{2,\text{cm}})$ (the "seesaw model" from lecture). Given some $m_1 x_{1,\text{cm}}$, with $m_1 = 10M_\odot$, $m_2(x_{2,\text{cm}})$ can take the same value either for

a lower mass than m_1 and a longer $x_{1,\text{cm}}$, or (as in our case) a higher $m_2 = 10M_\odot$ and a lower $x_{2,\text{cm}}$.

Note: Incidentally, we know that $x_{1,\text{cm}} + x_{2,\text{cm}} = a$, where $a = 1000\text{AU}$ is the distance between the two stars. Using the previous equation, we can solve for $x_{1,\text{cm}} = (m_2/m_1)x_{2,\text{cm}}$, and substitute in to actually find their values:

$$\begin{aligned} x_{1,\text{cm}} + x_{2,\text{cm}} &= \frac{m_2}{m_1}x_{2,\text{cm}} + x_{2,\text{cm}} = \left(1 + \frac{m_2}{m_1}\right)x_{2,\text{cm}} = a \\ x_{2,\text{cm}} &= \frac{a}{10M_\odot/1M_\odot + 1} = \frac{a}{11} = 91\text{ AU} \\ x_{1,\text{cm}} &= a - x_{2,\text{cm}} = \frac{10a}{11} = 909\text{ AU} \end{aligned} \tag{4}$$

(b) Using Newton's modification of Kepler's laws, **compute the orbital period** of the system.The orbital period in this case is just

$$T_{\text{orb}} = 2\pi\left(\frac{a^3}{G(M_1 + M_2)}\right)^{1/2} = 2.383 \times 10^6\,\text{y} \tag{5}$$

Assuming that $M_2 \ll M_1$, as we did for planets, would give us an answer that is off by several percent, $T_{\text{orb}} = 2.500 \times 10^6\,\text{y}$. The closer in mass M_1 and M_2 are, the greater the discrepancy.

(c) **Which star has a longer lifespan**, and why? Scaling off the mass-luminosity relation $L \propto M^4$, **compute the main-sequence lifetime** of each star. Recall that the Sun has a main-sequence lifetime of 10 billion years.The 1 $M_\odot$ star has a longer lifetime, because as in the previous worksheet, the lifetime scales as $\tau \propto M^{-3}$. Applying this scaling, the $1M_\odot$ star will live for 10 billion years, while the $10M_{\text{odot}}$ star will live for $\tau_* = (10)^{-3}\tau_\odot = 10$ million years.

(d) **Order and describe the evolutionary phases** *(protostar, giant, main sequence, white dwarf, etc.)*, from beginning to end, for the

i. 1 $M_\odot$ star

1. protostar—cloud of gas contracting until fusion begins
2. main sequence—fusion of hydrogen and helium.
3. giant—hydrogen and helium fusion throughout star. Ends in planetary nebula.
4. White dwarf—small, hot remnant of stellar core, supported by electron degeneracy pressure

ii. 10 $M_\odot$ star

1. protostar
2. main sequence
3. supergiant—star fuses heavier and heavier elements in onion layers
4. supernova—core collapses as described earlier. Leaves neutron star or black hole ($\gtrsim 40M_\odot$).

For each star, **which stage** accounts for the majority of its total lifespan?
The main sequence, in both cases, accounts for the majority of the star's active lifespan.

4. *Challenge Problem* When a star dies it releases its internal gravitational energy as light, $E_{\text{supernova}} \approx GM_*^2/R_*$. The typical star that goes supernova has a mass $M_* \approx 20M_\odot$ and radius $R_* \approx 200R_\odot$. Assuming that this energy is released over the course of roughly 1 hour, **compute the typical luminosity** of a supernova. **How close to Earth** would a supernova have to explode to be dangerous to life on Earth?